

# Chapter 1

## Fundamentals

For each exercise requiring a graph on the calculator, the work required to receive full credit is as follows, unless specifically changed in the directions of the problem, or by the instructor.

1. For each function graphed, show the function using the notation you have to enter in the calculator. Thus, the function is written appropriately on *one line* using parentheses,  $\wedge$  for exponents,  $/$  for division,  $\sqrt{\quad}$  for roots, etc. For example:

$$y1=x(x^3-5x^2+1)-100 \quad \text{or} \quad y1=(2x-5)/(3x+2) \quad \text{or} \quad y1 = \sqrt{x^4 - 3}$$

2. Show an appropriate window which does not use a lot of extra whitespace, using either  $[-2, 5]$  by  $[-8, 7]$  notation for  $[x\text{-min}, x\text{-max}]$  by  $[y\text{-min}, y\text{-max}]$ , or label the “ends” of the  $x$ - and  $y$ -axes. The  $x$ -scale and  $y$ -scale are assumed to be one unless otherwise specified.
3. Show the graph (fairly decently sketched).
4. If zeros, local extrema, intersections, etc. are needed, show the values before rounding, using at least six decimal places when possible. Then **round your final answer** as indicated in the problem.
5. When interval notation is appropriate, use it.

Here are a couple of examples to illustrate the work that is required on paper for graded homework unless indicated otherwise by your instructor.

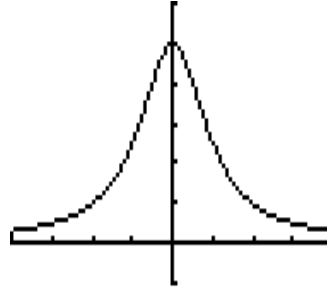
1) Graph  $f(x) = \frac{5}{x^2 + 1}$ .

2) Find the  $x$ -intercepts for  $f(x) = 0.2x^4 - 0.7x^3 + 2x - 1.6$ . Then find all values of  $x$  for which  $f(x) < 0$ , rounding to three decimal places.

1) Graph  $f(x) = \frac{5}{x^2 + 1}$ .

Calculator:

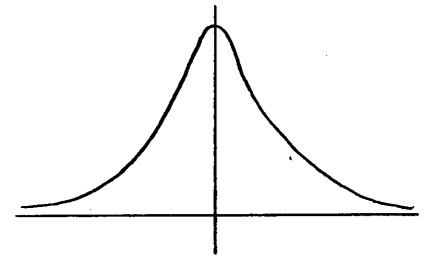
$y_1 = 5/(x^2 + 1)$   $[-4, 4]$  by  $[-1, 6]$



On paper:

window shown using  $[x, x]$  by  $[y, y]$  notation

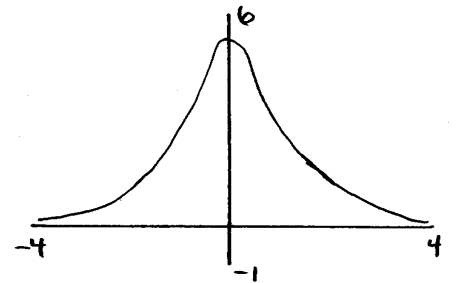
$$y_1 = 5/(x^2 + 1)$$
$$[-4, 4] \text{ by } [-1, 6]$$



or

window shown by labeling the ends of the axes

$$y_1 = 5/(x^2 + 1)$$



- 2) Find the  $x$ -intercepts for  $f(x) = 0.2x^4 - 0.7x^3 + 2x - 1.6$ .  
Find all values of  $x$  for which  $f(x) < 0$ .

Calculator:

$$y_1 = .2x^4 - .7x^3 + 2x - 1.6$$

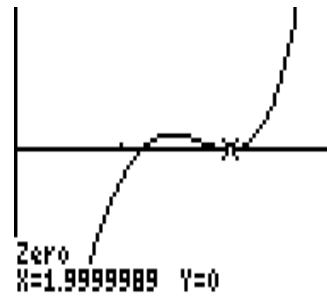
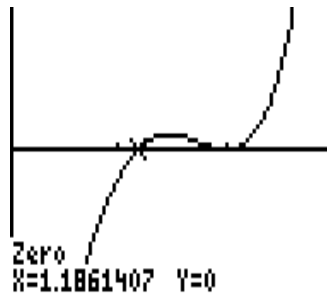
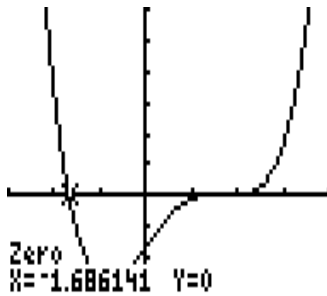
$[-3, 4]$  by  $[-3, 6]$



Finding  $x$ -intercepts

$[-3, 4]$  by  $[-3, 6]$

$[0, 3]$  by  $[-0.5, 0.5]$

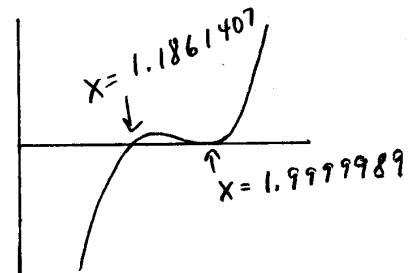
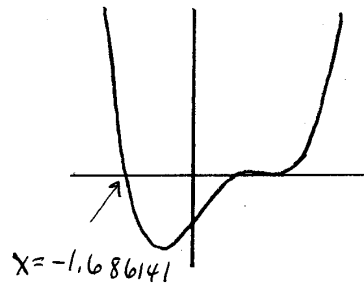


On paper:

$$y_1 = .2x^4 - .7x^3 + 2x - 1.6$$

$[-3, 4]$  by  $[-3, 6]$

$[0, 3]$  by  $[-.5, .5]$



$x$ -intercepts are  $-1.686, 1.186, 2.0$

$f(x) < 0$  for  $x$  in  $(-1.686, 1.186)$



Example, simplify  $16^{-\frac{3}{4}}$ , without a calculator (by hand), and with a calculator.

$$\text{Solution by hand } 16^{-\frac{3}{4}} = \left(\frac{1}{16}\right)^{\frac{3}{4}} = \left(\sqrt[4]{\frac{1}{16}}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\text{Solution with a calculator, } 16 \wedge (-3/4) = 0.125$$

Exercises Simplify each expression.

For exercises 1 - 12:

- a) Do each exercise without a calculator, showing some intermediate work.  
(It is important that you are able to do these without a calculator.)
- b) Do each exercise with a calculator, showing
- the expression you enter into the calculator,
  - the answer using at least six decimal places (when they exist),
  - the final answer rounded to three decimal places.

1. $32^{\frac{3}{5}}$	2. $27^{\frac{2}{3}}$	3. $\left(\frac{16}{81}\right)^{\frac{3}{4}}$
4. $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$	5. $\frac{4^8}{4^6}$	6. $\frac{2^4}{2^{-2}}$
7. $81^{\frac{3}{4}} \cdot 4^{-\frac{3}{2}}$	8. $16^{-\frac{5}{4}} \cdot 64^{\frac{2}{3}}$	9. $\frac{27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}}{27^3}$
10. $\frac{64^{-\frac{2}{3}} \cdot 64^{\frac{7}{3}}}{64^2}$	11. $\left(\frac{4}{9}\right)^{-\frac{3}{2}}$	12. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

Exercises 13 - 16, use a calculator, showing

- the expression you enter into the calculator,
- the answer using at least six decimal places (when they exist),
- the final answer rounded to three decimal places.

For #15, see page 68 for exponential functions.

13. $\frac{\sqrt{\pi + \pi^2}}{3}$	14. $\sqrt{\pi + \pi^{\frac{2}{3}}}$	15. $e^{\frac{\pi\sqrt{67}}{3}}$	16. $\pi^{\frac{e\sqrt{5+\pi}}{3}}$
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## 1.3 Algebraic Expressions

Read section 0.2 (Graphing Basics and Polynomial Functions, page 11) before working these exercises.

Review section 0.1 (Entering Expressions Correctly, page 8) as needed.

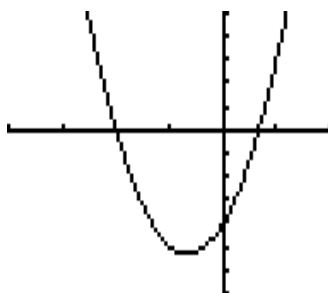
Factor each expression, if possible, without a calculator. If factorization is possible, then graph the original expression and your factored expression on the same graph to verify they are the same. If the expression is not factorable, no graph is necessary.

For example, factor  $3x^2 + 4x - 4$  to get  $(3x - 2)(x + 2)$ .

Now graph  $y_1 = 3x^2 + 4x - 4$  and  $y_2 = (3x - 2)(x + 2)$ .

Suppose we use a window of  $[-4, 2]$  by  $[-7, 5]$ .

```
Plot1 Plot2 Plot3
\Y1=3X^2+4X-4
\Y2=(3X-2)(X+2)
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



The graphs are identical - one is on top of the other. Thus, our factoring is correct.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

### Exercises

Factor each expression, if possible, without a calculator. If factorization is possible, then graph the original expression and the factored expression on the same graph to verify they are the same. If the graphs are different, find your mistake in the factoring or in the way you entered the expressions into the calculator and fix it. Your graphs must match.

1.  $x^2 + x - 12$

2.  $3x^2 - 6x - 24$

3.  $3x^2 - 4x - 4$

4.  $2x^2 + x - 6$

5.  $9x^2 - 16$

6.  $4x^2 - 25$

7.  $x^4 + 1$

8.  $4x^2 + 1$

9.  $x^3 - 8$

10.  $8x^6 + 27$

11.  $3x^3 - x^2 + 3x - 1$

12.  $6x^3 + 3x^2 - 4x - 2$

13.  $3x^{\frac{2}{3}} - 2x^{\frac{1}{3}}$

14.  $2x^{-\frac{5}{2}} - \frac{3}{2}x^{\frac{3}{2}}$

## 1.4 Rational Expressions

Read section 0.3.1 (Rational Functions, page 21) before working these exercises. Review section 0.1.3 (How to Enter Expressions, page 10) as needed.

Simplify each expression algebraically. Then graph the original expression and the simplified expression to verify they are the same.

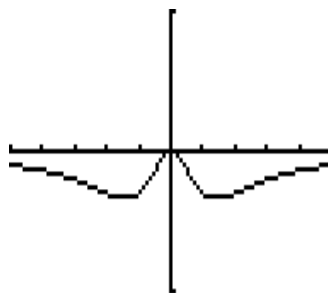
$$\begin{aligned} \text{For example, } \frac{1}{x^2 + 1} - \frac{4}{x^2 + 4} &= \frac{1(x^2 + 4)}{(x^2 + 1)(x^2 + 4)} - \frac{4(x^2 + 1)}{(x^2 + 4)(x^2 + 1)} \\ &= \frac{x^2 + 4 - 4(x^2 + 1)}{(x^2 + 1)(x^2 + 4)} = \frac{x^2 + 4 - 4x^2 - 4}{(x^2 + 1)(x^2 + 4)} \\ &= \frac{-3x^2}{(x^2 + 1)(x^2 + 4)} \end{aligned}$$

Graph  $y_1 = \frac{1}{x^2 + 1} - \frac{4}{x^2 + 4}$  and  $y_2 = \frac{-3x^2}{(x^2 + 1)(x^2 + 4)}$  with a window of  $[-5, 5]$  by  $[-1, 1]$ .

```

Plot1 Plot2 Plot3
\Y1=1/(X^2+1)-4/
(X^2+4)
\Y2=(-3X^2)/((X^
2+1)(X^2+4))
\Y3=
\Y4=
\Y5=

```



We see that both functions produce the same graph (one is on top of the other). Thus, our simplified expression is the same as the original expression.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

### Exercises

Simplify each expression algebraically. Then graph the original expression and the simplified expression to verify they are the same. If the graphs are different, find your mistake in simplifying, or in graphing, and fix it so the graphs match.

1.  $\frac{8x^2 + 16x}{4x^2}$
2.  $\frac{9x^5 - 18x}{3x^3}$
3.  $\frac{3}{x - 2} - \frac{1}{2 - x}$
4.  $\frac{2}{1 - x} - \frac{3}{x - 1}$
5.  $\frac{1}{6x} + \frac{2}{5x} + \frac{4}{x}$
6.  $\frac{1}{2x} + \frac{3}{4x} - \frac{2}{3x}$
7.  $\frac{4}{x^2 + 2} - \frac{2}{x^2 + 3}$
8.  $\frac{3}{x^2 + 1} - \frac{4}{x^2 + 2}$

## 1.5 Equations

Read section 0.9 (Solving Equations by Graphing, page 124) and section 0.3.4 (Radical Functions, page 38) before working these exercises.

Review section 0.2.2 (Polynomial Functions, page 12) and section 0.3.1 (Rational Functions, page 21) as needed.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

### Exercises

In exercises 1 - 10:

- a) Solve the equation algebraically, using exact values for the answer (no calculator).
- b) Solve the equation graphically, rounding to three decimal places.

It is strongly suggested that you use the  $x$ -intercept method for some of the problems and the points of intersection method for other problems, so you practice both methods.

Caution: When finding the  $x$ -intercept and entering a lower or upper bound, the  $x$ -value entered must be in the domain of the function. Thus, if  $x = 0$  is not in the domain (as in #5 and #6), we cannot enter  $x = 0$  as a bound.

1.  $3x + 2 - 5(x + 1) = 6x + 4$

2.  $5(x + 3) + 4x - 5 = -(2x - 4)$

3.  $6x^2 + 7x = 3$

4.  $3x^2 = -2x + 5$

5.  $3 - \frac{4}{x} = \frac{2}{x^2}$

6.  $\frac{3}{x} - \frac{2}{x^2} = -4$

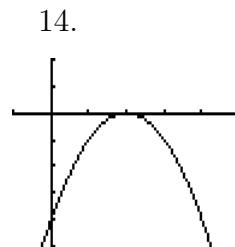
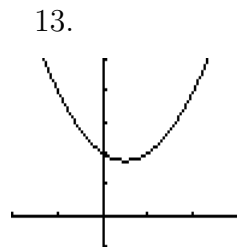
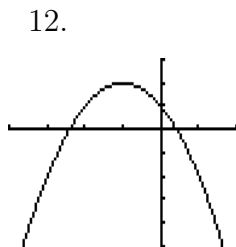
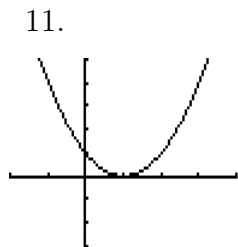
7.  $\sqrt{2}x - \sqrt{x + 15} = 0$

8.  $x - \sqrt{15 - 2x} = 0$

9.  $\sqrt{8x + 1} + 4 = 2x - 1$

10.  $\sqrt{4x + 5} - 2 = 2x - 7$

For each graph shown in exercises 11 - 14, determine if the discriminant for the equation graphed is *negative*, *zero*, or *positive*, and explain your reasoning.



In exercises 15 - 20, graph the function given. By looking at the graph, determine if the discriminant is *positive*, *negative*, or *zero*, and explain your reasoning.

15.  $y = x^2 - 2x - 1$

16.  $y = 2x^2 - 3x + 1$

17.  $y = -x^2 - 4x - 4$

18.  $y = x^2 - 4x + 4$

19.  $y = -2x^2 - x - 1$

20.  $y = -2x^2 + 3x - 2$

For exercises 21 and 22, read section 0.3.2 (Absolute Value Functions, page 28). Use the absolute value function in your function definition for the graph - do not remove the absolute value signs algebraically.

21. Given  $x^4 - 2.1x^3 + 3.5x^2 - 4.7x - 3.1 = |2.4x - 3.7|$ , (round to three decimal places)

(a) solve for  $x$  by using the  $x$ -intercept method

(b) solve for  $x$  by using the points of intersection method

22. Given  $-x^3 - 1.2x^2 + 0.75x - 3.8 = |-3.8x + 2| - 1$ , (round to three decimal places)

(a) solve for  $x$  by using the  $x$ -intercept method

(b) solve for  $x$  by using the points of intersection method

## 1.6 Modeling with Equations

Review section 0.9 (Solving Equations by Graphing, page 124) as needed.

A calculator hint: If you receive an error when entering a function such as  $x(x-1)(2x+1)$ , you need to use the multiplication symbol, such as  $x*(x-1)(2x+1)$ , especially on the TI-89.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

### Exercises

Define the variable, write the needed equation, find the domain for the variable, think about possible values for  $y$  in order to determine an appropriate window, then solve the equation by graphing, and state the answer in words.

(Use either  $x$ -intercept method or points of intersection method.)

1. Dalton's company is expanding their current line of products to produce open top cardboard boxes. Their first order is for a box whose volume is 2860 cubic inches. The design specifications call for starting with a rectangular piece of cardboard where the length is 3 times the width. Squares measuring 4 inches on a side are cut out of each corner of cardboard piece and the remaining sides are folded up to form the box. What should the dimensions of the original piece of cardboard be in order to make this box?
2. The volume of a box of crackers is  $102.68 \text{ in}^3$ . The length of the box is 3.6 inches more than the width, and its depth is 2.21 inches. Find the length and width of the box to the nearest hundredth.
3. A cylindrical thermos has a surface area of  $273 \text{ in}^2$ . Its height is 9.4 inches. What is the radius of the circular top, rounded to the nearest tenth? (The surface area is  $2\pi rh + 2\pi r^2$ , for radius  $r$  and height  $h$ .)
4. Lee has a rectangular plot to use for a flower garden. She plans to include a gravel border of uniform width around the flower garden so visitors can walk completely around the flowers to view them more easily. The total size of the garden and gravel border is to be 15 feet by 21 feet. If Lee has enough gravel to cover 117 square feet for the border, how wide will the gravel border be? What will the dimensions of the actual flower garden be? Round answers to nearest tenth.
5. The local parks department decides to construct a triangular sidewalk around an odd shaped monument. The sidewalk will have the shape of a right triangle with the longer leg 20 yd shorter than the hypotenuse, and the other leg 30 yd shorter than the longer leg. Find the total length of the sidewalk, rounding each side to the nearest whole number.

## 1.7 Inequalities

Read section 0.10 (Solving Inequalities by Graphing, page 147) and section 0.3.2 (Absolute Value, page 28) before working these exercises.

Review section 0.1.2 (Other Roots and Rational Expressions, page 9) and section 0.3.1 (Rational Functions, page 21) as needed.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

### Exercises

Solve each inequality by graphing, rounding to the nearest hundredth.  
Write each solution in interval notation.

(It is suggested that you do some exercises using the  $x$ -intercept method and others using the points of intersection method in order to practice each technique.)

1.  $4.1 - 3.5x < -1 - 7.5x$

2.  $x^2 + 5x + 5 > 2$

3.  $x^2 < x + 1$

4.  $x^2 > 8$

5.  $\sqrt[3]{x} < x$

6.  $x^3 + 11x \leq 6x^2 + 6$

7.  $\frac{x+2}{x+3} \geq \frac{x-1}{x-2}$

8.  $\frac{4.9}{x} \leq x$

9.  $(x - 1.8)^2 > (x + 1.5)^2$

10.  $(x + 1.2)^2 < (x - 1.3)^2$

11.  $\left| \frac{x+1}{2} \right| \geq 5$

12.  $3.2 - |1.7x + 4| \leq 2.4$

13.  $\left| 1 - \frac{x}{2} \right| < 0.05$

14.  $\left| 1 + \frac{x}{2} \right| < 0.05$

15. One of the properties for  $n^{\text{th}}$  roots states that when  $n$  is even,  $\sqrt[n]{x^n} = |x|$ . Let's check this out by doing some graphing, using a viewing window of  $[-5, 5]$  by  $[-5, 5]$ . Graph each part separately, one-at-a-time, and sketch the graph on paper so you can compare the graphs.

a) Graph  $y = |x|$ .

b) Graph  $y = \sqrt{x^2}$ .

c) Graph  $y = x$ .

Which graphs (thus, which functions) are the same?

## 1.8 Coordinate Geometry

Read section 0.3.3 (Circles, page 33) before working these exercises.

Review section 0.2.2 (Polynomial Functions, page 12), section 0.3.1 (Rational Functions, page 21) as needed.

Recall that we can only graph  $y = \dots$ ; thus, you will need to solve for  $y$  before proceeding with the graph on some of the exercises.

Refer to page 187 for an example of the work required on paper for all graded homework unless directed otherwise by your instructor.

### Exercises

Determine if the equation is symmetric with respect to the  $x$ -axis,  $y$ -axis, or origin, or none of these, by graphing and making your decision from the graph. Be sure to draw the complete graph, joining the curve at the  $x$ -axis if appropriate. (Hint: for circles, put the equation in standard form, then solve for  $y$ .)

1.  $y = \frac{4x^2}{x^2 + 1}$

2.  $y = \frac{x^2 + 1}{x^4 + 2}$

3.  $y^2 = \frac{5}{x^2}$

4.  $y = x^2 - 3x - 1$

5.  $y^2 = x$

6.  $y^2 = x^2 - 1$

7.  $x^2 + y^2 = 5$

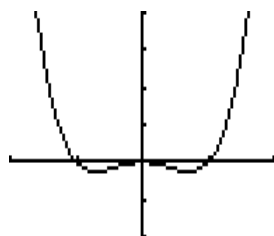
8.  $x^2 + y^2 = 8$

9.  $x^2 + y^2 + 6y = 5$

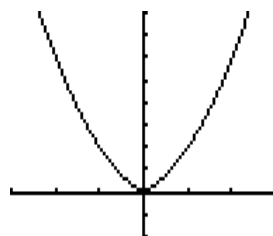
10.  $x^2 + y^2 - 2y + 1 = 3$

Determine from the graph if each equation is symmetric with respect to the  $x$ -axis,  $y$ -axis, or origin, or none of these.

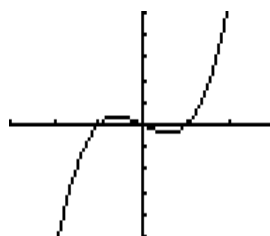
11.



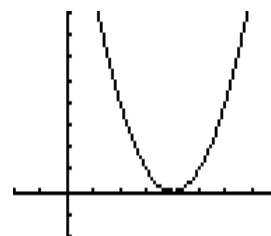
12.



13.



14.



15. Which of the following equations might have the graph shown? More than one answer is possible. Explain why the equation can, or cannot, be the graph shown. Do this without graphing the equation on your calculator. (Hint: Compare the circle equation's center and radius to the graphed circle's center and radius. What do we know about the graph's center? What do we know about the graphed circle's radius compared to the distance of the center from the origin?)

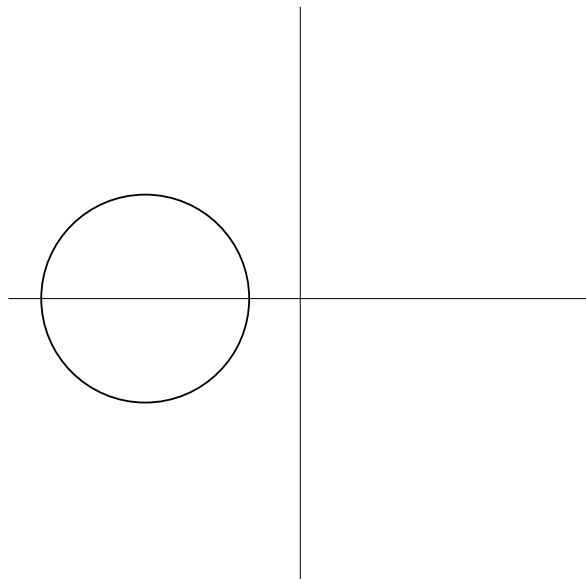
(a)  $(x - 2)^2 + y^2 = 3$

(b)  $(x + 2)^2 + y^2 = 3$

(c)  $x^2 + (y - 2)^2 = 3$

(d)  $x^2 + y^2 + 10x + 16 = 0$

(e)  $x^2 + y^2 + 10x - 2y = 1$

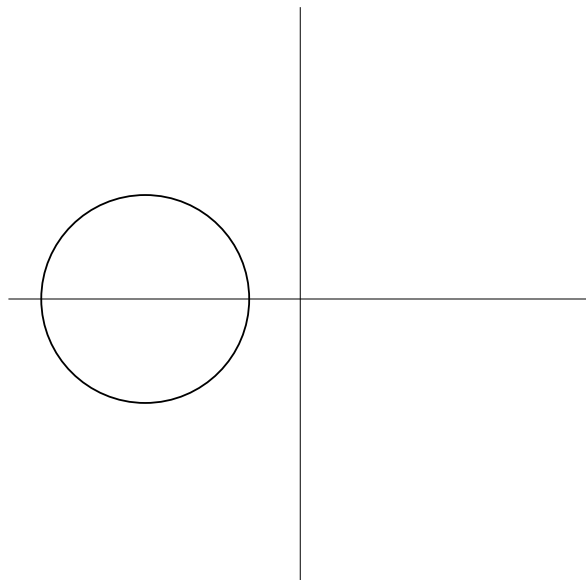


16. Which of the following equations might have the graph shown? More than one answer is possible. Explain why the equation can, or cannot, be the graph shown. Do this without graphing the equation on your calculator. (Refer to the hint in #15.)

(a)  $(x + 2)^2 + y^2 = 4$

(b)  $x^2 + y^2 + 9x + 10 = 0$

(c)  $x^2 + y^2 - 9x - 10 = 0$



## 1.9 Solving Equations and Inequalities Graphically

An idea we might consider is answering questions about a family of equations.

Read section 0.4.1 (Graphing a Family of Functions, page 46) and 0.4.2 (Solutions for a Family of Equations, page 54) before working these exercises.

### Exercises

1. For the family of equations  $x^3 - 3x = k$ 
  - (a) Draw the graphs of  $y_1 = x^3 - 3x$  and  $y_2 = k$  for  $k = -3, -2, 1, 2, 4$ , on the same coordinate system. (So, we are looking for points of intersection.)  
To graph  $y_2 = k$  for  $k = -3, -2, 1, 2, 4$ , enter  $\{-3, -2, 1, 2, 4\}$  as the formula for  $y_2$ .
  - (b) How many solutions of the equation  $x^3 - 3x = k$  are there for each value of  $k$ ?
  - (c) For what range of values of  $k$  does the equation have one solution? two solutions? three solutions? (Use interval notation where appropriate.)
  
2. For the family of equations  $-x^2 - 2x = k$ 
  - (a) Draw the graphs of  $y_1 = -x^2 - 2x - k$  for  $k = -3, -1, 0, 1, 3$ , on the same coordinate system. (Here we are looking for  $x$ -intercepts.)  
OR  
Draw the graphs of  $y_1 = -x^2 - 2x$  and  $y_2 = k$  for  $k = -3, -1, 0, 1, 3$ , on the same coordinate system. (Here we are looking for points of intersection.) (Remember,  $y = 0$  is the  $x$ -axis, so we will not see it as it is drawn on the graph.)
  - (b) How many solutions of the equation  $-x^2 - 2x = k$  are there for each value of  $k$ ?
  - (c) For what range of values of  $k$  does the equation have no solution? one solution? two solutions? (Use interval notation where appropriate.)
  
3. For the family of equations  $|x - 1| = k$ 
  - (a) Draw the graphs of  $y_1 = |x - 1|$  and  $y_2 = k$  for  $k = -4, -2, 0, 2, 4$ , on the same coordinate system.
  - (b) How many solutions of the equation  $|x - 1| = k$  are there for each value of  $k$ ?
  - (c) For what range of values of  $k$  does the equation have no solution? one solution? two solutions? (Use interval notation where appropriate.)

4. For the family of equations  $|x - k| - 1 = 0$

- (a) Draw the graphs of  $y_1 = |x - k| - 1$  for  $k = -4, -2, 0, 2, 4$  on the same coordinate system.
- (b) How many solutions of the equation  $|x - k| = 1$  are there for each value of  $k$ ?
- (c) For what range of values of  $k$  does the equation have no solution? one solution? two solutions? (Use interval notation where appropriate.)

## 1.10 Lines

Read/review section 0.4.1 (Graphing a Family of Functions, page 46) before working exercise 1.

### Exercises

1. Graph the family of lines on the same coordinate system. What do the lines have in common? As  $k$  increases, what changes do you see in the graph? (Does the slant of the line change, if so how? Does the line move left/right, up/down?)

(a)  $y = kx + 2$  for  $k = -4, -2, -\frac{1}{2}, 0, \frac{1}{2}, 2, 4$

(b)  $y = 2x + k$  for  $k = -4, -2, 0, 2, 4$

2. Match each equation with the most likely graph. Do not make any assumptions about the scale, except it is the same on all graphs. Explain your reasoning for each choice.

a)  $y = -2x + 1$

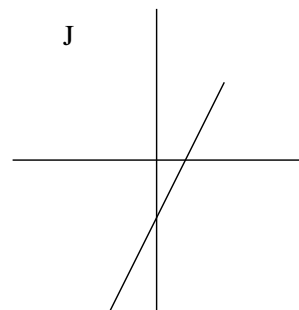
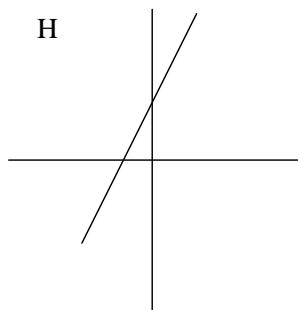
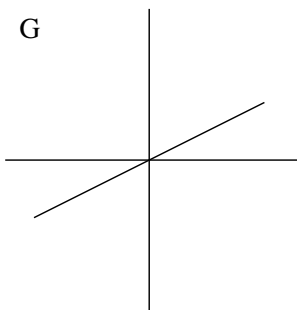
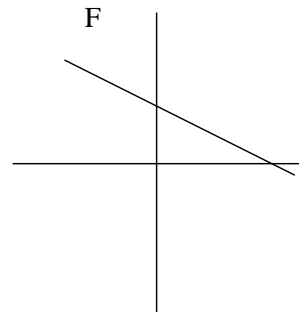
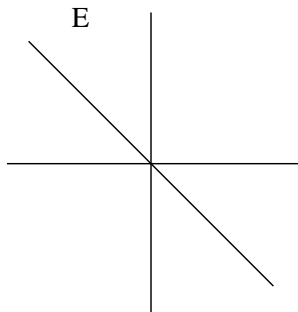
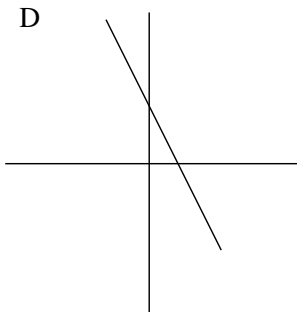
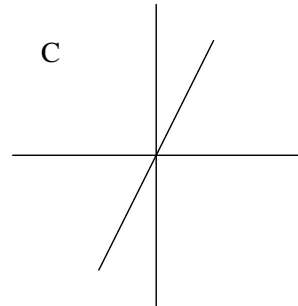
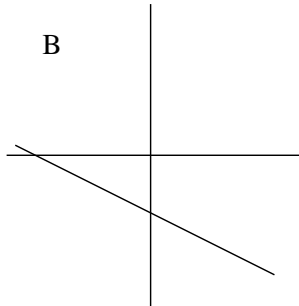
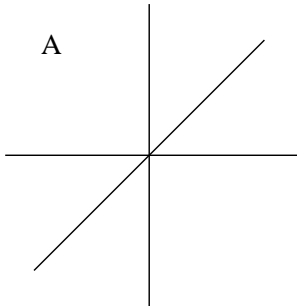
b)  $y = 2x$

c)  $y = 2x + 1$

d)  $y = x$

e)  $y = -\frac{1}{2}x + 1$

f)  $y = \frac{1}{2}x$



3. The figure shows the graph of two parallel lines. Which of the following pairs of equations might have such a graph? Explain the reasoning for your decision. (You must decide this without drawing the graphs for the equations.)

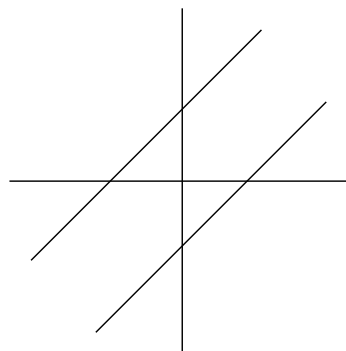
A)  $x - 2y = 3$   
 $x + 2y = 7$

B)  $x - y = -2$   
 $2x - 2y = -4$

C)  $x + y = 2$   
 $x + y = -1$

D)  $x + 2y = 2$   
 $x + 2y = -1$

E)  $x - y = -2$   
 $x - y = 1$



4. The figure shows the graph of two perpendicular lines. Which of the following pairs of equations might have such a graph? Explain the reasoning for your decision. (You must decide this without drawing the graphs for the equations.)

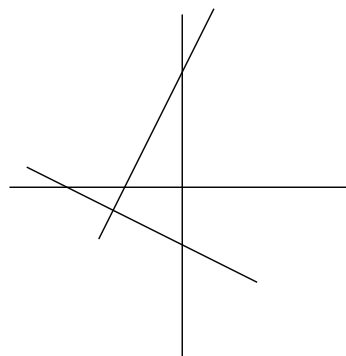
A)  $y - 2x = 2$   
 $y + 2x = -1$

B)  $y - 2x = 2$   
 $x + 2y = -1$

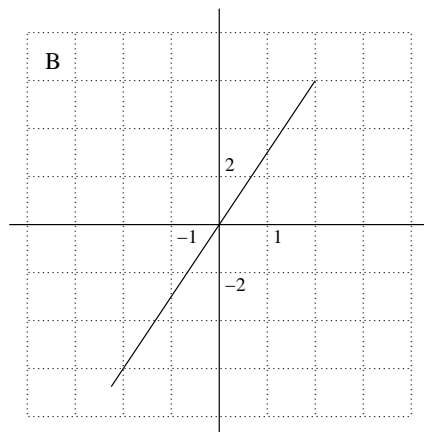
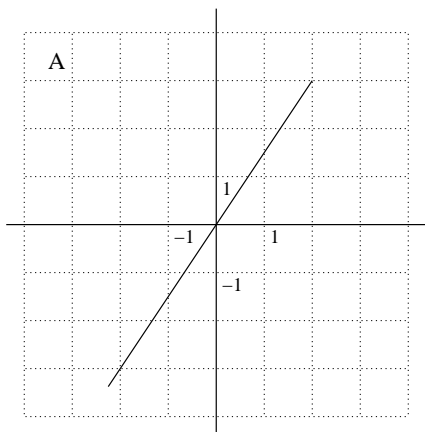
C)  $y - 2x = 0$   
 $2y + x = 0$

D)  $2x + y = -2$   
 $2y + x = -2$

E)  $2y - x = 2$   
 $2y + x = -2$



5. Given these two graphs, which line has the larger slope?



## 1.11 Modeling Variation

See textbook for exercises.