0.4 Family of Functions/Equations

By a *family of functions*, we are referring to a function definition such as

$$f(x) = mx + 2 \text{ for } m = -2,\ -1,\ -\tfrac{1}{2},\ 0,\ \tfrac{1}{2}, 1,\ 2.$$

This says, work with all the functions obtained by letting m equal the values given.

Often we are asked to graph a *family of functions* in the same viewing window in order to determine what the functions have in common, or to see how changing one part of the function will affect the graph. We may also use a family of functions to learn more about the solutions of an equation.

0.4.1 Graphing a Family of Functions

Let's graph $f(x) = mx + 2$ for $m = -2,\ -1,\ -\frac{1}{2},\ 0,\ \frac{1}{2}, 1,\ 2.$

This means we need to graph 7 different functions:

$f(x) = -2x + 2$	$f(x) = .5x + 2$
$f(x) = -1x + 2$	$f(x) = x + 2$
$f(x) = -.5x + 2$	$f(x) = 2x + 2$
$f(x) = 0x + 2$ which is $f(x) = 2$	

We could graph each of the 7 functions on our calculator by entering 7 different definitions. However, there is a very nice shortcut for a "family of functions" which we want to learn.

As you type in the function definition, in place of m, use a list which shows the values of m, such as $\{-2, -1, -.5, 0, .5, 1, 2\}$.

So the first function $f(x) = mx + 2$ looks like

$$y = \{-2, -1, -.5, 0, .5, 1, 2\}x + 2$$

The graphs will be drawn in the order we list the values for m; thus, it helps if we list the values in either decreasing order or increasing order.

TI-83 (see page 47), TI-89 (see page 49), TI-86 (see page 51).

TI-83: Family of Functions

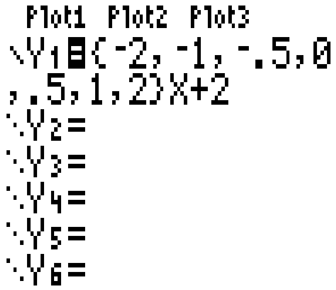
Let's graph $f(x) = mx + 2$ for $m = -2,\ -1,\ -\frac{1}{2},\ 0,\ \frac{1}{2}, 1,\ 2.$

Press [Y=] and clear any function left over from a previous graph.

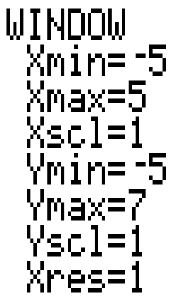
To enter the formula $\mathtt{y_1} = \{-2, -1, -.5, 0, .5, 1, 2\}\mathtt{x} + 2$, do the following.

The braces { and } are second functions on the left parenthesis and the right parenthesis keys. The comma is to the left of the parentheses (above the [7]).

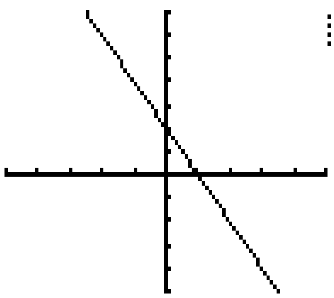
So, enter [2nd] and [{] `-2,-1,-.5,0,.5,1,2` [2nd] and [}] `x+2` to see

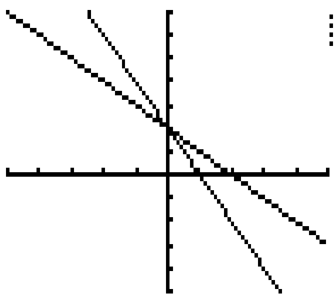


Press [Window] and enter values for $[-5, 5]$ by $[-5, 7]$.

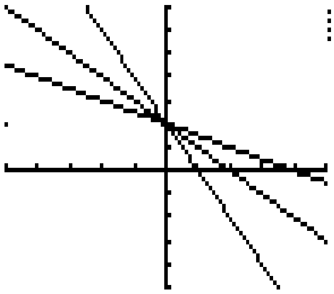


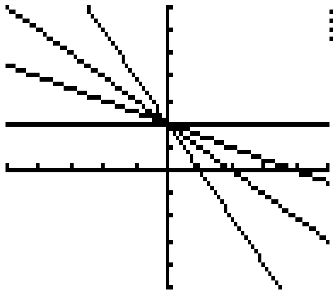
Press [Graph] and the functions are shown in the order we specified in our list for m.
First we have $m = -2$ and $y = -2x + 2$, then $m = -1$ and $y = -x + 2$.



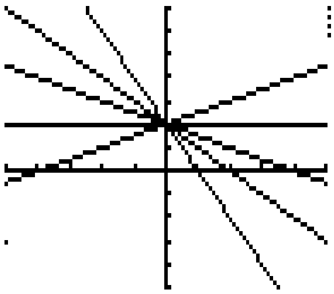


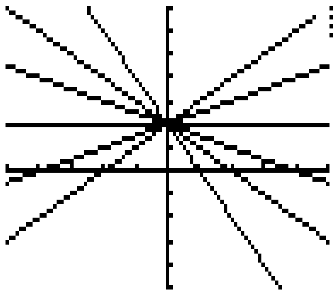
Then $m = -.5$ and $y = -.5x + 2$, $m = 0$ and $y = 2$.



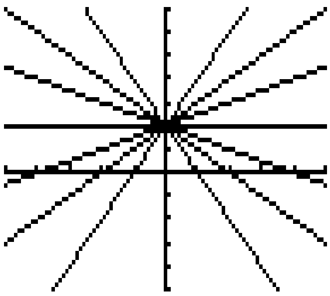


Next $m = .5$ and $y = .5x + 2$, $m = 1$ and $y = x + 2$.





And finally, $m = 2$ and $y = 2x + 2$.



Notice what these graphs have in common. They all have the same $y-$intercept of $y = 2$. Does this make sense from the formula $y = mx + 2$? Why?

As the slope m increased from -2 to 2, the lines progressed from slanting steep downward to flattening out to slanting steep upward (reading the graph from left to right).

See conclusion on page 53.

TI-89: Family of Functions

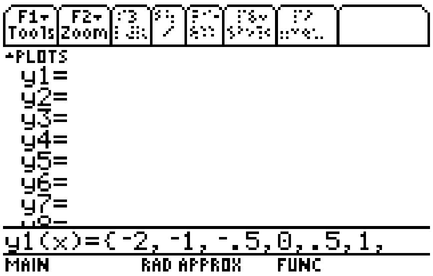
Let's graph $f(x) = mx + 2$ for $m = -2,\ -1,\ -\frac{1}{2},\ 0,\ \frac{1}{2}, 1,\ 2.$

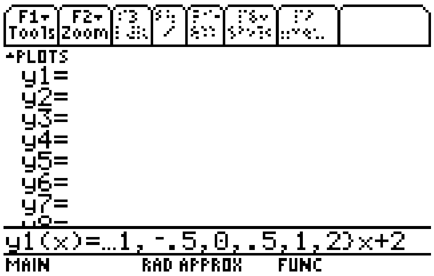
Press the green ◆ key and F1 Y= and clear any previous function.

To enter the formula $\mathtt{y}_1 = \{-2, -1, -.5, 0, .5, 1, 2\}\mathtt{x} + 2$, do the following.

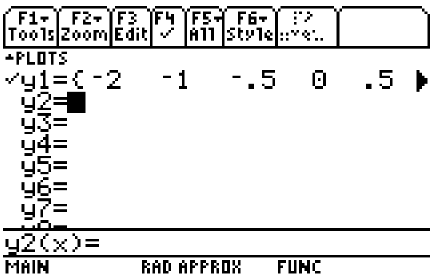
The braces { and } are second functions with the left parenthesis and the right parenthesis. The comma is to the right of the parentheses (above the 9).

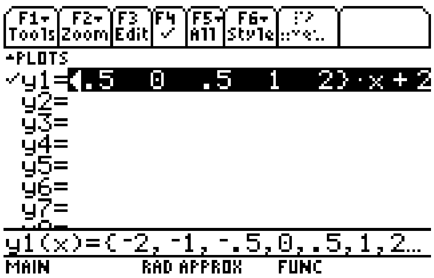
So, enter 2nd and { `-2,-1,-.5,0,.5,1,2` then 2nd and } `x+2` (The function is too long for one screen width, so it is shown in two pieces.)



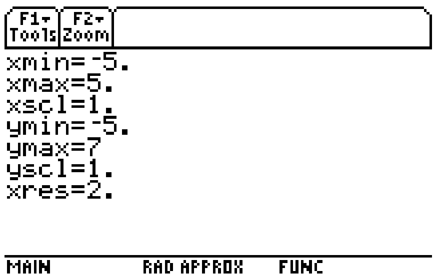


Press Enter and verify the formula is correct (left picture). To see the complete formula, you need to move the cursor up to definition y_1 and use the right arrow key (right picture).

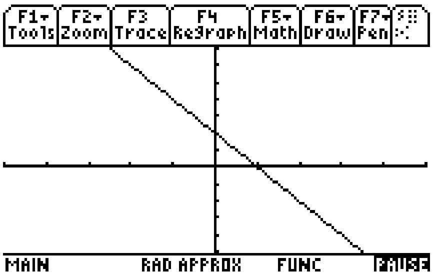


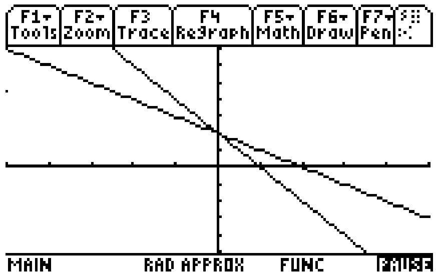


Press ◆ and F2 Window and enter values for $[-5, 5]$ by $[-5, 7]$.

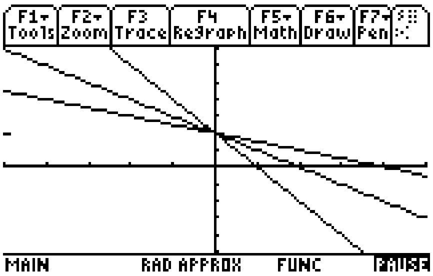


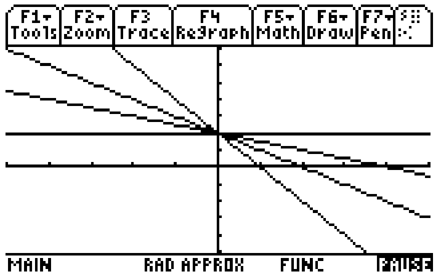
Press ♦ and F3 Graph and the functions are shown in the order we specified in our list for m. First we have $m = -2$ and $y = -2x + 2$, then $m = -1$ and $y = -x + 2$.



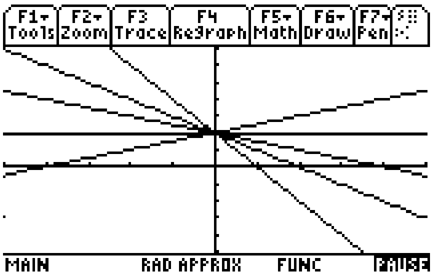


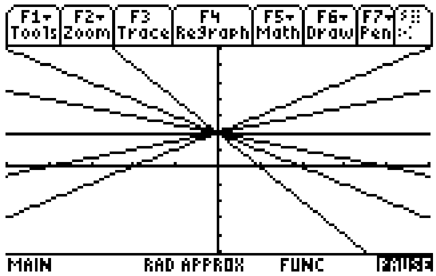
Then $m = -.5$ and $y = -.5x + 2$, $m = 0$ and $y = 2$.



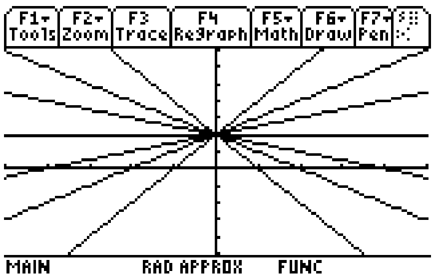


Next $m = .5$ and $y = .5x + 2$, $m = 1$ and $y = x + 2$.





And finally, $m = 2$ and $y = 2x + 2$.



Notice what these graphs have in common. They all have the same $y-$intercept of $y = 2$. Does this make sense from the formula $y = mx + 2$? Why?

As the slope m increased from -2 to 2, the lines progressed from slanting steep downward to flattening out to slanting steep upward (reading the graph from left to right).

See conclusion on page 53.

TI-86: Family of Functions

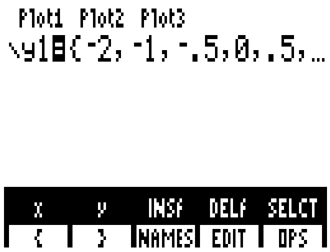
Let's graph $f(x) = mx + 2$ for $m = -2,\ -1,\ -\frac{1}{2},\ 0,\ \frac{1}{2}, 1,\ 2$.

Press Graph , then F1 y(x)= , and clear any functions previously defined.

To enter the formula $\mathtt{y_1} = \{-2, -1, -.5, 0, .5, 1, 2\}\mathtt{x} + 2$, do the following.

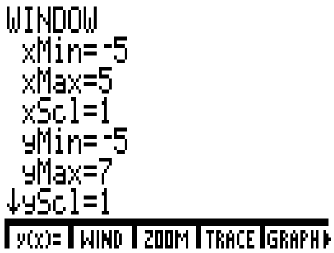
Find the braces { and } by pressing 2nd and List (`List` is the second function for subtraction). The comma is to the left of 4 .

Press F1 to get { and enter `-2,-1,-.5,0,.5,1,2` followed by F2 to get }. Then enter `x+2` and the definition is finished. Press Exit to get rid of the list options. (The function is shown here using 2 pictures since the definition is wider than one screen.)



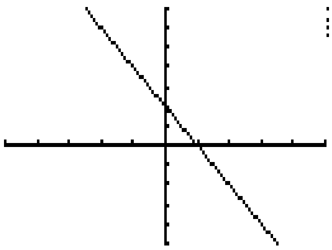
Press Exit to leave the `List` menu.

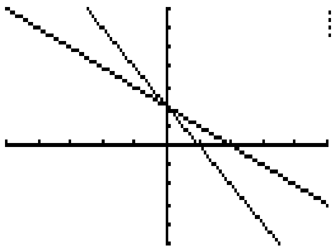
Press 2nd and F2 Wind and enter values for $[-5, 5]$ by $[-5, 7]$.



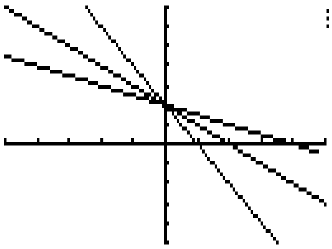
Press F5 Graph and the functions are shown in the order we specified in our list for m.

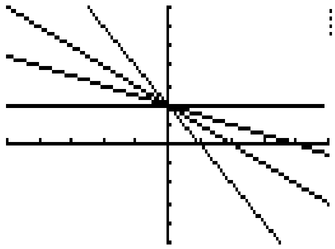
First we have $m = -2$ and $y = -2x + 2$, then $m = -1$ and $y = -x + 2$.



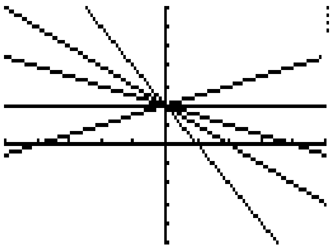


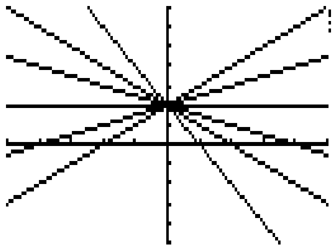
Then $m = -.5$ and $y = -.5x + 2$, $m = 0$ and $y = 2$.



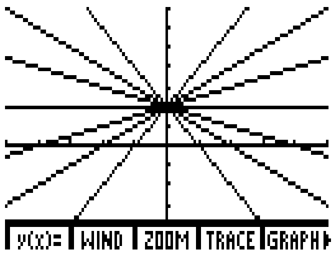


Next $m = .5$ and $y = .5x + 2$, $m = 1$ and $y = x + 2$.





And finally, $m = 2$ and $y = 2x + 2$.



Notice what these graphs have in common. They all have the same $y-$intercept of $y = 2$. Does this make sense from the formula $y = mx + 2$? Why?

As the slope m increased from -2 to 2, the lines progressed from slanting steep downward to flattening out to slanting steep upward (reading the graph from left to right).

Conclusion on the next page.

Conclusion for family of functions

Whenever we need to graph a family of functions, such as

$$f(x) = x^3 + c \text{ for } c = 0,\ \pm\frac{1}{2},\ \pm 1,\ \pm 2,$$

we put a list in the function definition in place of c as follows:

$$\texttt{y}_1 = \texttt{x} \wedge 3 + \{-2, -1, -.5, 0, .5, 1, 2\}$$

This definition stands for

$f(x) = x^3 - 2$	$f(x) = x^3 + .5$
$f(x) = x^3 - 1$	$f(x) = x^3 + 1$
$f(x) = x^3 - .5$	$f(x) = x^3 + 2$
$f(x) = x^3$	

0.4.2 Solutions for a Family of Equations

Consider the family of equations $x^2 - 2x + 1 = k$. Suppose we are asked to determine the number of solutions for $k = -4, 0, 4$, and then the range of values of k for which the equation has no solutions, one solution, and two solutions.

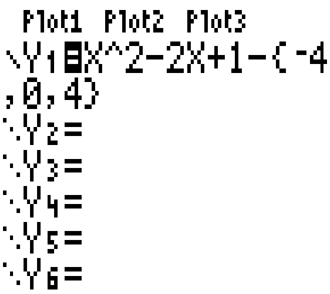
If we use the $x-$intercept method of solving this equation (see section 0.9.1), we set the equation equal to 0: $\quad x^2 - 2x + 1 - k = 0$

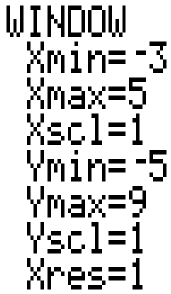
Then graph $y = x^2 - 2x + 1 - k$ for the given values of $k = -4, 0, 4$.

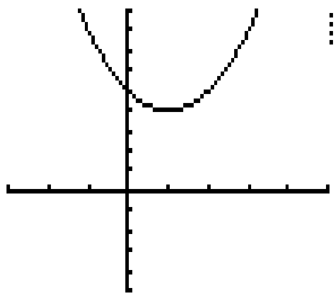
The method is illustrated here using the TI-83. The appropriate graphing techniques previously discussed for the TI-89 and TI-86 will produce similar graphs, and the same final results as shown here.

Press [Y=] and clear any functions from a previous graph.

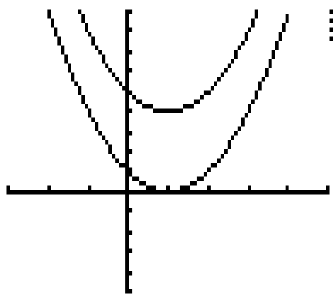
Enter $\mathtt{x^2 - 2x + 1 - \{-4, 0, 4\}}$ (left picture). Press [Window] and set the viewing window for $[-3, 5]$ by $[-5, 9]$ (middle picture). Press [Graph] to see $y = x^2 - 2x + 1 - (-4)$, or $y = x^2 - 2x + 5$ (right picture).

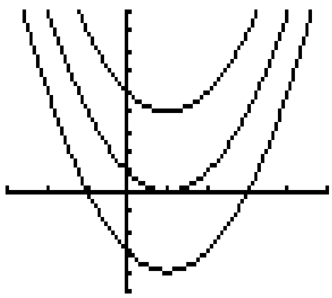






Then we see $y = x^2 - 2x + 1 - (0)$, or $y = x^2 - 2x + 1$ (left picture). And finally $y = x^2 - 2x + 1 - (4)$, or $y = x^2 - 2x - 3$ (right picture).

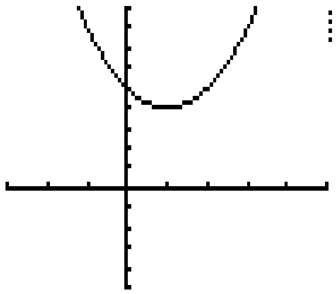




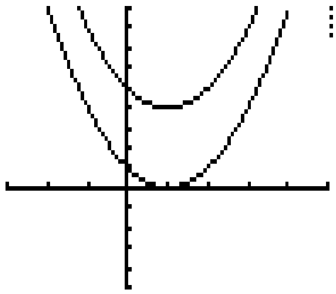
Initially, we were asked to determine two things. So, let's answer both questions.

- the number of solutions for $x^2 - 2x + 1 = k$ when $k = -4, 0, 4$.

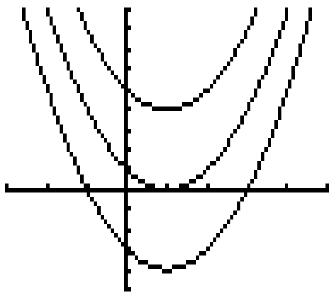
  When $k = -4$, there are *no solutions* since there are no $x-$intercepts.

  When $k = 0$, there is *one solution* since the vertex of the parabola is on the $x-$axis, giving us one $x-$intercept. The value appears to be $x = 1$. You should test this value in the equation to verify that the graph actually does intersect the $x-$axis at $x = 1$.

  When $k = 4$, there are *two solutions* since we have two $x-$intercepts.

- the range of values of k for which the equation has no solutions, one solution, and two solutions.

  First, notice $x^2 - 2x + 1$ can be factored as $(x-1)^2$, so we have $(x-1)^2 = k$.

  When $k = 0$, the vertex is on the $x-$axis and we have one $x-$intercept. Thus, the equation $x^2 - 2x + 1 = k$ or $(x-1)^2 = 0$ has *one solution*,

  When $k < 0$, the graph of the parabola is shifted up vertically and we have no $x-$intercepts; thus, the equation $x^2 - 2x + 1 = k$ or $(x-1)^2 = k$ has *no solutions.* (In our case, we had $(x-1)^2 = -4$ and we know a squared quantity is never equal to a negative number. We also recognize that $(x-1)^2 + 4 = y$ shifts the graph up vertically, giving no $x-$intercepts.)

  When $k > 0$, the graph of the parabola is shifted down vertically and we have two $x-$intercepts; thus, the equation $x^2 - 2x + 1 = k$ or $(x-1)^2 = k$ has *two solutions.* (In our case, we had $(x-1)^2 = 4$ which we know has two solutions. We also recognize that $(x-1)^2 - 4 = y$ shifts the graph down vertically, giving 2 $x-$intercepts.)