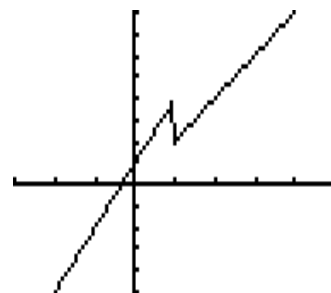


0.5 Graphing Piecewise-Defined Functions

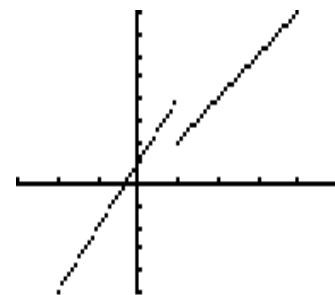
To graph a piecewise-defined function, such as $f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

we must specify each piece of the function and the values of x to use for that particular piece.

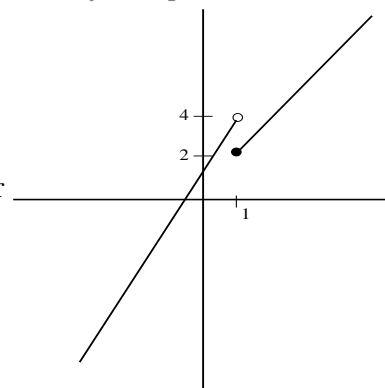
If we graph the function in the normal *connected* mode, all the points plotted will be connected by a line, such as you see here. This means the piecewise-defined function is graphed incorrectly - the left and right pieces are joined when they should not be connected unless they have a common endpoint.



Thus, we must use *dot* mode to see the correct graph. However, keep in mind that the calculator will not show what happens at the “endpoints” of $(1, 4)$ and $(1, 2)$. There is often a gap between the two pieces with the endpoints shown as an open circle \circ (representing a hole in the graph since the point is *not* part of the graph) and a closed circle \bullet (representing a point that *is* part of the graph).



Thus, you must clearly indicate on paper whether each endpoint is included, using \bullet , or not included, using \circ . You should also label the x - and y -axes at these endpoints. So your final graph on paper should look like this:



TI-83 (see page 57), TI-89 (see page 59), TI-86 (see page 62).

Instructions for functions with three-piece definitions begin on page 64.

TI-83: Piecewise-defined Functions

To graph $f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

we need to enter $y_1 = (3x + 1)(x < 1) + (2x)(x \geq 1)$.

This statements means:

graph $y_1 = 3x + 1$ when $x < 1$ “and” graph $y_1 = 2x$ when $x \geq 1$.

Press **Y=** and then **Clear** if a function is left over from a previous graph.

Enter $(3x+1)(x$ (left picture).

To find the $<$ symbol, press **2nd** and **Test** (middle picture).
(Test is the second function for Math.)

Select $5:<$ by either typing 5, or using the arrow keys and pressing **Enter** (right picture).

```

Plot1 Plot2 Plot3
\Y1=(3X+1)(X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
  
```

```

TEST LOGIC
1:=
2:≠
3:>
4:≥
5:<
6:≤
  
```

```

Plot1 Plot2 Plot3
\Y1=(3X+1)(X<
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
  
```

Continue typing $) + (2x)(x$ (left picture).

Find \geq by pressing **2nd** and **Test**. This time choose $4: \geq$ (right picture).

```

Plot1 Plot2 Plot3
\Y1=(3X+1)(X<1)+
(2X)(X
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
  
```

```

Plot1 Plot2 Plot3
\Y1=(3X+1)(X<1)+
(2X)(X≥
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
  
```

Finish typing 1) (left picture).

Press **Window** and set the viewing window of $[-3, 5]$ by $[-5, 8]$ (middle picture).

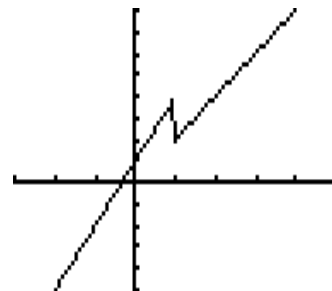
Press **Graph** and we see this graph (right picture) since the calculator is still in *connected* mode.

```

Plot1 Plot2 Plot3
Y1=(3X+1)(X<1)+
(2X)(X≥1)
Y2=
Y3=
Y4=
Y5=
Y6=
    
```

```

WINDOW
Xmin=-3
Xmax=5
Xscl=1
Ymin=-5
Ymax=8
Yscl=1
Xres=1
    
```



To change to *dot* mode, using the method from page 22, or from section 0.8.1 page 100, press **y=** to return to the function definition (left picture).

Using the left arrow key, move the cursor to the backslash to the left of y_1 . Press **Enter** 6 times to change the backslash to a set of 3 dots (middle picture).

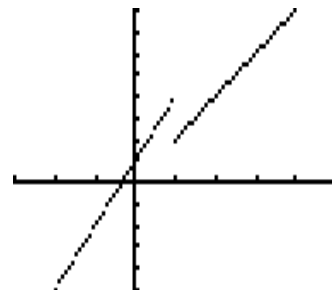
Press **Graph** to redraw the function using *dot* mode. Notice the gap is correct now between the endpoints (right picture).

```

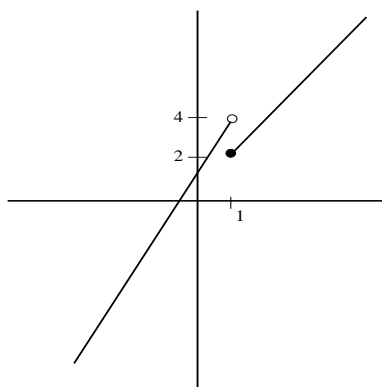
Plot1 Plot2 Plot3
Y1=(3X+1)(X<1)+
(2X)(X≥1)
Y2=
Y3=
Y4=
Y5=
Y6=
    
```

```

Plot1 Plot2 Plot3
Y1=(3X+1)(X<1)+
(2X)(X≥1)
Y2=
Y3=
Y4=
Y5=
Y6=
    
```



Clearly indicating what happens at the endpoints, and labeling the x - and y -axes for the endpoints, we have this graph when we sketch it on paper.



TI-89: Piecewise-defined Functions

To graph the function $f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

we need to tell the calculator this: $y_1 = \text{when}(x < 1)$ graph $y = (3x + 1)$ and for all other values of x , graph $y = 2x$. On the calculator, it looks like $y_1 = \text{when}(x < 1, 3x + 1, 2x)$

Press the green \blacklozenge key and $\boxed{\text{F1 Y=}}$ and $\boxed{\text{Clear}}$ to erase any previous functions.

To enter the formula, we need to type `when` so press $\boxed{\text{Catalog}}$ then type `w` to get the list of functions beginning with `w`. The first one is `when(` (left picture).

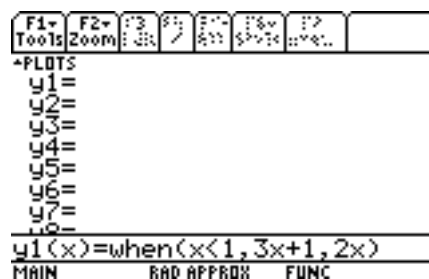
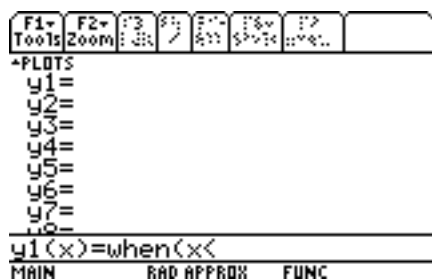
Press $\boxed{\text{Enter}}$ (right picture).



An alternate version of entering `when` is to press $\boxed{2\text{nd}}$ and $\boxed{\text{a-lock}}$ to lock it in *alpha* mode. Type `when` then press $\boxed{\text{alpha}}$ to turn off the alpha lock. Then type in the left parenthesis $\boxed{(}$.

Enter `x < 1, 3x + 1, 2x)` to finish the definition. Type `x` then to find the `<` use $\boxed{2\text{nd}}$ and $\boxed{<}$ which is the second function on $\boxed{0}$ (left picture).

Finish the expression `1, 3x+1, 2x)` using the comma $\boxed{,}$ as needed (right picture). (Comma is above $\boxed{9}$.)



Press **Enter** and you see the piecewise definition of the function (left picture). This is actually a true-false statement which says if $x < 1$ is true, graph $y_1 = 3x + 1$, otherwise (meaning $x < 1$ is false, so when $x \geq 1$), graph $y_1 = 2x$.

Press **◆**, **F2 Window** and enter the appropriate values for $[-3, 5]$ by $[-5, 8]$ (right picture).

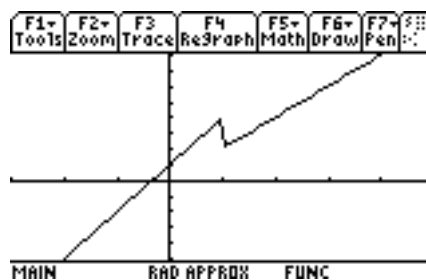
```

F1- F2- F3- F4- F5- F6- F7-
Tools Zoom Edit ✓ All Style ...etc..
+PLOTS
✓y1={ 3·x+1,x < 1
      2·x,else
y2=
y3=
y4=
y5=
y6=
y2(x)=
MAIN RAD APPROX FUNC
  
```

```

F1- F2-
Tools Zoom
xmin=-3.
xmax=5.
xsc1=1.
ymin=-5.
ymax=8.
ysc1=1
xres=2.
MAIN RAD APPROX FUNC
  
```

Press **◆** and **F3 Graph** to see the graph

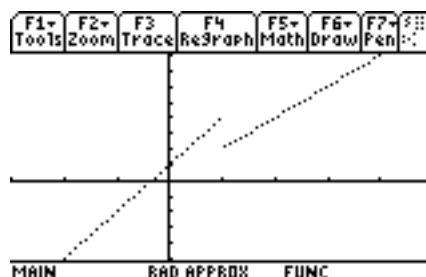


Since the calculator is in *connected* mode, the lines are joined between the left piece and the right piece, where there should be a gap. We must change to *dot* mode. Return to the definition of the function by pressing the green **◆** key and **F1 Y=**. Move the cursor to the $y_1 =$ definition and press **2nd** and **F6 Style**. We are currently in *Line* mode (see the checkmark? This is *connected* on other calculators). Select 2:Dot by entering 2.

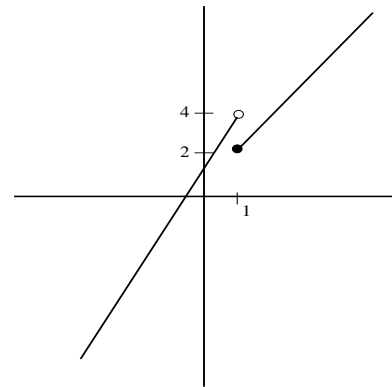
Press **◆** and **F3 Graph** to draw the graph again. Notice the gap between the two lines.

```

F1- F2- F3- F4- F5- F6- F7-
Tools Zoom Edit ✓ All Style ...etc..
+PLOTS
✓y1={ 3·x+1,x < 1
      2·x,else
y2=
y3=
y4=
y5=
y6=
y1(x)=when(x<1,3*x+1,2*x)
TYPE OR USE ↔↑+ (CENTER) OR (ESC)
  
```



Clearly indicating what happens at the endpoints, and labeling the x - and y -axes for the endpoints, we get this graph when we sketch it on paper:



For future reference, the inequality symbols can be found in several ways. Two possibilities are shown.

All four symbols $<$, $>$, \leq , \geq can be found using **2nd** and **Math** (the second function on key 5, then select **8:Test** to see the inequality symbols).

OR

symbol	keystrokes
$<$	2nd , < found with 0
$>$	2nd , > found with .
\leq	Catalog , Z , scroll down until you find \leq
\geq	Catalog , Z , scroll down until you find \geq

TI-86: Piecewise-defined Functions

To graph the piecewise defined function, $f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

we need to enter $y_1 = (3x + 1)(x < 1) + (2x)(x \geq 1)$.

This statements means:

graph $y_1 = 3x + 1$ when $x < 1$ “and” graph $y_1 = 2x$ when $x \geq 1$.

Press **Graph**, then **F1** to select $y(x) =$. Press **Clear** to erase any other functions defined.

Enter $(3x+1)(x$ (left picture).

To find the $<$ symbol, press **2nd** and **Test** (middle picture). (**Test** is the second function for 2.)

Press **F2** to select $<$ (right picture).

```
Plot1 Plot2 Plot3
√y1(3 x+1)(x
```

```
Plot1 Plot2 Plot3
√y1(3 x+1)(x
```

```
Plot1 Plot2 Plot3
√y1(3 x+1)(x<
```

```
MODE WIND ZOOM TRACE GRAPH
x y INSP DELF SELCT▶
```

```
x y INSP DELF SELCT
= < > ≤ ≥▶
```

```
x y INSP DELF SELCT
= < > ≤ ≥▶
```

Continue typing $1) + (2x)(x$ (left picture).

Notice the inequality options for **Test** are still available (middle picture).

Press **F5** for \geq .

Finish typing $1) (x \geq 1)$ (right picture).

Note that the line does not “wrap” as we type. Instead it scrolls to the right.

```
Plot1 Plot2 Plot3
√y1(1)(x<1)+(2 x)(x
```

```
Plot1 Plot2 Plot3
√y1(1)(x<1)+(2 x)(x≥
```

```
Plot1 Plot2 Plot3
√y1(1)(x<1)+(2 x)(x≥1)
```

```
x y INSP DELF SELCT
= < > ≤ ≥▶
```

```
x y INSP DELF SELCT
= < > ≤ ≥▶
```

```
x y INSP DELF SELCT
= < > ≤ ≥▶
```

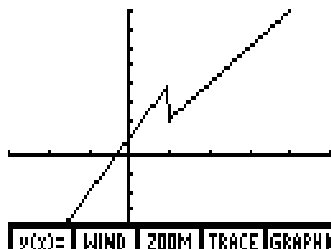
Press **Exit** to get out of the **Test** options.

Set the viewing window for $[-3, 5]$ by $[-5, 8]$ (left picture).

Press **F5 Graph** and we see this graph (right picture) since the calculator is still in *connected* mode.

```

WINDOW
xMin=-3
xMax=5
xScl=1
yMin=-5
yMax=8
↓yScl=1
v(x)= | WIND | ZOOM | TRACE | GRAPH▶
    
```



To change to *dot* mode, using the method from page 27, or from section 0.8.1 page 104, press **F1 y(x)=** to return to the function definition (left picture).

With the cursor on the $y_1 =$ definition, press **More** to see additional options. Press **F3 Style** 6 times, to change the backslash to the left of y_1 so it shows 3 dots (middle picture).

Press **2nd** and **F5 Graph** to redraw the function using *dot* mode (right picture).

Notice the gap is correct now between the endpoints.

```

Plot1 Plot2 Plot3
y1=...x<1)+(2 x)(x≥1)
    
```

```

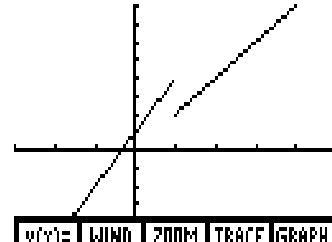
Plot1 Plot2 Plot3
y1=(3x+1)(x<1)+(2x)...
    
```

```

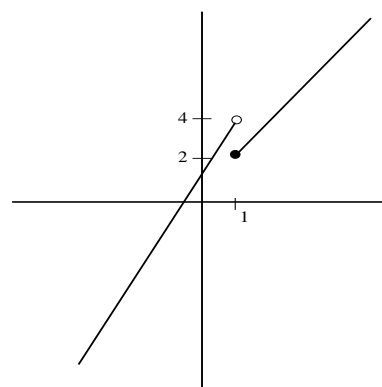
x  y  INSF  DELF  SELCT
=  <  >  ≡  ≧  ▶
    
```

```

v(x)= | WIND | ZOOM | TRACE | GRAPH▶
ALL+ | ALL- | STYLE |
    
```



Clearly indicating what happens at the endpoints, and labeling the x - and y -axes for the endpoints, we get this graph when we sketch it on paper:



A Three-part Piecewise-defined Function

Now, suppose we need to graph a function with three parts, such as

$$f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } 1 \leq x < 2 \\ -x & \text{if } x \geq 2 \end{cases}$$

How do we enter this function?

TI-83 (this page), TI-89 (see page 65), TI-86 (see page 66).

TI-83: Three-part Functions

Enter the function as

$$y_1 = (3x + 1)(x < 1) + (2x)(1 \leq x)(x < 2) + (-x)(x \geq 2)$$

Recall, $<$, \leq , $>$, \geq can be found using **2nd** and **Test**.

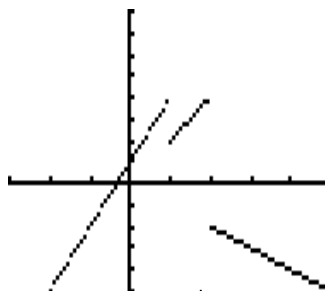
```

Plot1 Plot2 Plot3
\Y1=(3X+1)(X<1)+
(2X)(1≤X)(X<2)+(-
X)(X≥2)
\Y2=
\Y3=
\Y4=
\Y5=
    
```

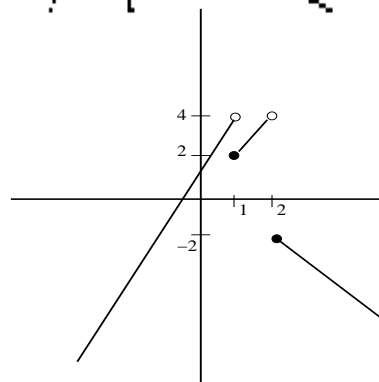
There is no shortcut for the middle portion $(2x)(1 \leq x)(x < 2)$.

We cannot combine the inequalities $(1 \leq x)(x < 2)$ and still get a correct graph.

Set a viewing window of $[-3, 5]$ by $[-5, 8]$ and draw the graph in *dot* mode.



When we sketch this on paper, we must add the appropriate endpoints of $(1, 4)$ with \circ , $(1, 2)$ with \bullet , $(2, 4)$ with \circ , $(2, -2)$ with \bullet , and label the scale, to see



TI-89: Three-part Functions

To graph

$$f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } 1 \leq x < 2 \\ -x & \text{if } x \geq 2 \end{cases}$$

we need to enter the function $y_1 = \text{when}(x < 1, 3x + 1, \text{when}(x < 2, 2x, -x))$ (shown in two pieces below since the definition is wider than the window).

Recall $<$ can be found using $\boxed{2\text{nd}}$ and $\boxed{<}$, the second function on $\boxed{0}$.

This function definition says “if $x < 1$, then use $y = 3x + 1$; otherwise if $x < 2$ (meaning $x \geq 1$ and $x < 2$), then use $y = 2x$; otherwise (meaning $x \geq 2$), use $y = -x$.”

```

F1+  F2+  F3  F4  F5+  F6+  F7+  F8+
Tools Zoom  Del  /  <  >  >>  >>>
+PLOTS
y1=
y2=
y3=
y4=
y5=
y6=
y7=
y8=
-----
y1(x)=when(x<1,3x+1,when(x<
MAIN  RAD APPROX  FUNC
  
```

```

F1+  F2+  F3  F4  F5+  F6+  F7+  F8+
Tools Zoom  Del  /  <  >  >>  >>>
+PLOTS
y1=
y2=
y3=
y4=
y5=
y6=
y7=
y8=
-----
y1(x)=...1,when(x<2,2x,-x))
MAIN  RAD EXACT  FUNC
  
```

Press $\boxed{\text{Enter}}$ and we see this function for y_1 :

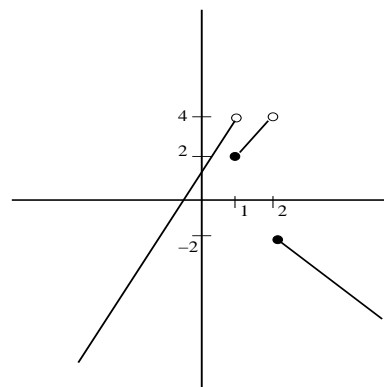
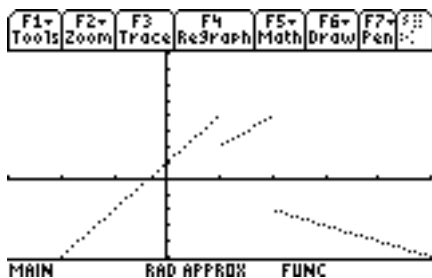
This looks confusing, but it says
 if $x < 1$, graph $y_1 = 3x + 1$,
 else we know $x \geq 1$, so within
 this interval of $[1, \infty)$,
 if $x < 2$, graph $y_1 = 2x$,
 else graph $y_1 = -x$.

```

F1+  F2+  F3  F4  F5+  F6+  F7+  F8+
Tools Zoom Edit  ✓  All  Style  >>>
+PLOTS
✓y1={
  {3·x+1,x<1
  {2·x,x<2
  {-x,else,else
y2=
y3=
y4=
y5=
-----
y2(x)=
MAIN  RAD APPROX  FUNC
  
```

Set a viewing window for $[-3, 5]$ by $[-5, 8]$ and draw the graph in *dot* mode (left picture).

When we sketch this on paper, we must add the appropriate endpoints of $(1, 4)$ with \circ , $(1, 2)$ with \bullet , $(2, 4)$ with \circ , $(2, -2)$ with \bullet , and label the scale, as shown in the right picture.



TI-86: Three-part Functions

To graph $f(x) = \begin{cases} 3x + 1 & \text{if } x < 1 \\ 2x & \text{if } 1 \leq x < 2 \\ -x & \text{if } x \geq 2 \end{cases}$

Enter the function as $y_1 = (3x + 1)(x < 1) + (2x)(1 \leq x)(x < 2) + (-x)(x \geq 2)$

To find the $<, \leq, >, \geq$ symbols, press **2nd** and **Test** (the second function for 2).
(The function is shown here in three pieces since the definition is wider than one line.)

```

Plot1 Plot2 Plot3
Y1=(3 X+1)(X<1)+(2
Y2=

```

```

Plot1 Plot2 Plot3
Y1=(3 X+1)(X<1)+(2X)(1<=X)(X<2)...
Y2=

```

```

Plot1 Plot2 Plot3
Y1=(3 X+1)(X<1)+(2X)(1<=X)(X<2)+(-X)(X<=2)
Y2=

```

```

X Y INSF DELF SELCT
= < > ≤ ≥

```

```

X Y INSF DELF SELCT
= < > ≤ ≥

```

```

X Y INSF DELF SELCT
= < > ≤ ≥

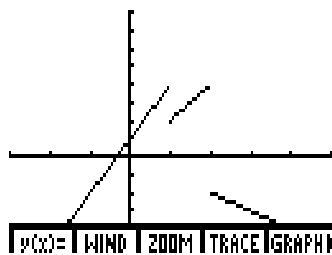
```

There is no shortcut for the middle portion $(2x)(1 \leq x)(x < 2)$.

We cannot combine the inequalities $(1 \leq x)(x < 2)$ and still get a correct graph.

Press **Exit** to get rid of the **Test** menu.

Set a viewing window of $[-3, 5]$ by $[-5, 8]$ and draw the graph in *dot* mode.



When we sketch this on paper, we must add the appropriate endpoints of $(1, 4)$ with \circ , $(1, 2)$ with \bullet , $(2, 4)$ with \circ , $(2, -2)$ with \bullet , and label the scale, to see

