

Section 4.1

1. (a) transformations

$g(x)$: right 1

$h(x)$: reflect y , up 2

$k(x)$: reflect x , down 1

(b) formulas:

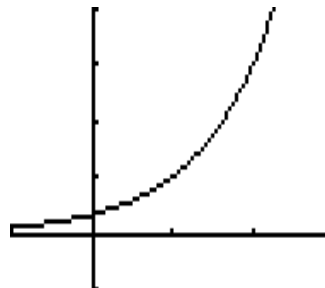
$$g(x) = 3^{x-1}$$

$$h(x) = 3^{-x} + 2$$

$$k(x) = -3^x - 1, \quad \text{or } k(x) = -1(3^x) - 1$$

(c) Be sure to draw the appropriate horizontal asymptotes.

(d) $g(x) = 3^{x-1}$ $y_1 = 3 \wedge (x - 1)$
 $[-1, 3]$ by $[-1, 4]$

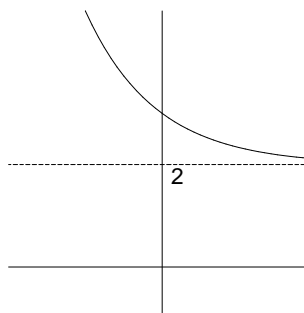


$h(x) = 3^{-x} + 2$ $y_1 = 3 \wedge (-x) + 2$
 $[-2, 3]$ by $[-1, 5]$

graph on calculator

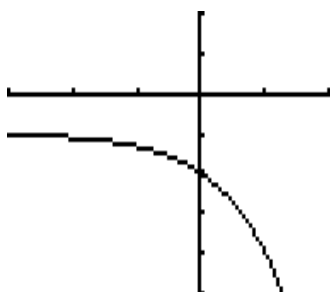


graph with asymptote

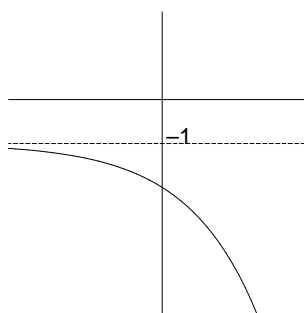


$k(x) = -3^x - 1$ $y_1 = -3 \wedge x - 1$
 $[-3, 2]$ by $[-5, 2]$

graph on calculator



graph with asymptote

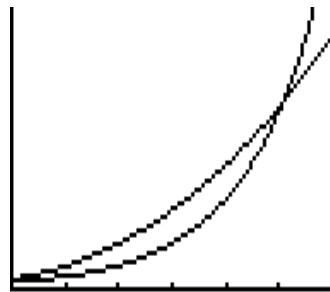
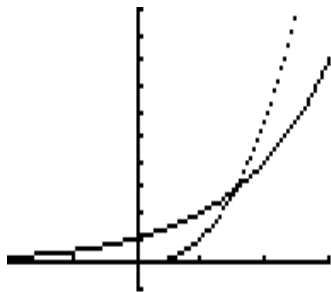


3. (a) $f(x) = 2^x$ grows faster as x gets very large

$$y_1 = 2 \wedge x \qquad y_2 = x \wedge 2.5 \quad \text{shown in dot mode}$$

$[-2, 3]$ by $[-1, 10]$, left graph shows $y_2 = x^{2.5}$ above $y_1 = 2^x$.

$[2, 8]$ by $[-1, 200]$, right graph shows the curves crossing a second time and $y_1 = 2^x$ is above $y_2 = x^{2.5}$.

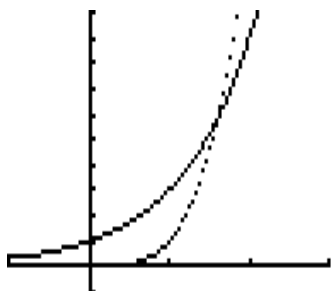


(b) $f(x) = 3^x$ grows faster as x gets very large

$$y_1 = 3 \wedge x \qquad y_2 = x \wedge 3.7 \quad \text{shown in dot mode}$$

$[-1, 3]$ by $[-1, 10]$, left graph shows $y_2 = x^{3.7}$ above $y_1 = 3^x$.

$[1, 8]$ by $[-5, 1200]$, right graph shows the curves crossing a second time and $y_1 = 3^x$ is above $y_2 = x^{3.7}$.

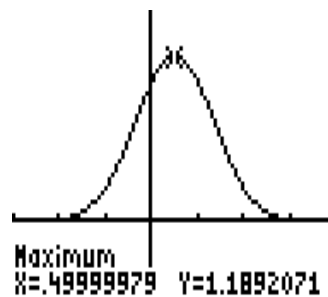


5. $y_1 = 2 \wedge (x - x \wedge 2)$ $[-3, 4]$ by $[-0.5, 1.5]$

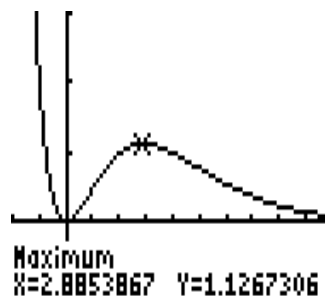
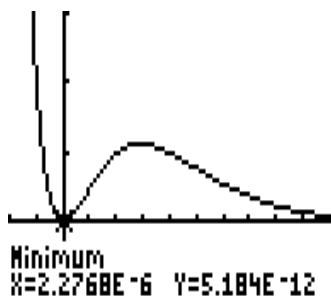
increasing: $(-\infty, .5]$ decreasing: $[.5, \infty)$

$y = 1.18920071$

maximum value is 1.19



7. $y_1 = x \wedge 2(2 \wedge (-x))$ $[-2, 10]$ by $[-1, 3]$



increasing: $[0, 2.89]$

decreasing: $(-\infty, 0] \cup [2.89, \infty)$

$y = 0$ (Recall, we represent $5.184E - 12$ as zero.)

minimum value is 0

$y = 1.1267306$

maximum value is 1.13

9. $y_1 = 1 - x \wedge 2 + 2 \wedge (-x)$ $[-5, 2]$ by $[-2, 3]$



We need to find out what happens between $x = -4$ and $x = -2$.

Since we are looking for the x -intercepts, we can consider something like $[-.5, .5]$ for the y -values to show what happens on the x -axis.

(Continued on next page.)

So, using $[-4, -2]$ by $[-.5, .5]$, we have



Zero
X=-3.407451 Y=0



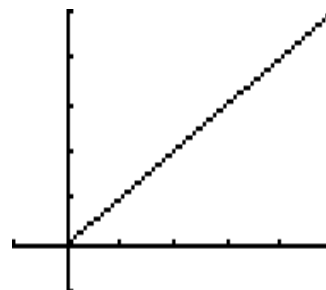
Zero
X=-3 Y=0

The real zeros are: -3.407451 , -3.00 , 1.1982502
To two decimal places, we have $x = -3.41$, -3 , and 1.20 .

Section 4.2

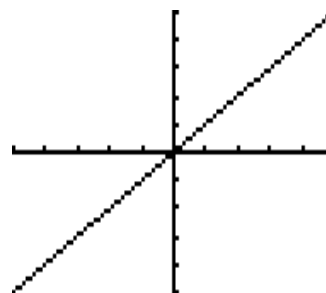
1. (a) domain $(0, \infty)$
 $[-1, 5]$ by $[-1, 5]$

```
Plot1 Plot2 Plot3
\Y1= e^(ln(X))
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



- (b) domain $(-\infty, \infty)$
 $[-5, 5]$ by $[-5, 5]$

```
Plot1 Plot2 Plot3
\Y1= ln(e^(X))
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



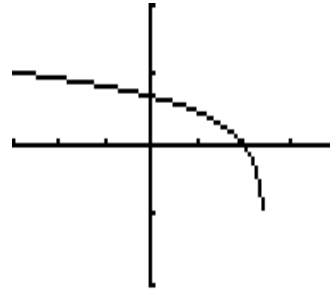
(c) $y = e^{\ln x} = x$, but the domain is $x > 0$ since the domain of $\ln x$ is $x > 0$. Thus, the graph of $y = x$ only exists in quadrant I.

$y = \ln(e^x) = x$, but the domain is $(-\infty, \infty)$ since the domain of $y = e^x$ is $(-\infty, \infty)$ and e^x yields only positive values for the natural logarithm to work with. Thus, the graph of $y = x$ exists for all real numbers x .

3. (a) Domain $(-\infty, \frac{5}{2})$

$y_1 = \log(5 - 2x)$ $[-3, 4]$ by $[-2, 2]$

vertical asymptote is $x = \frac{5}{2}$



3a. On paper:

$$f(x) = \log(5 - 2x)$$

$$5 - 2x > 0$$

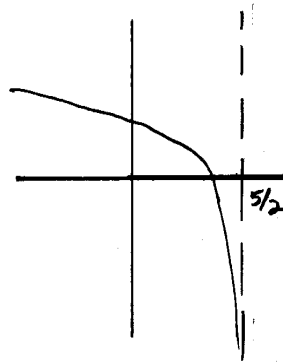
$$-2x > -5$$

$$x < \frac{5}{2}$$

$$\text{Domain } (-\infty, \frac{5}{2})$$

$$y_1 = \log(5 - 2x)$$

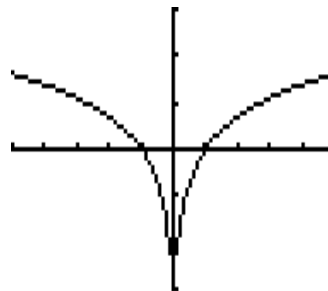
$$[-3, 4] \text{ by } [-2, 2]$$



(b) Domain $(-\infty, 0) \cup (0, \infty)$

$y_1 = \ln(\text{abs}(x))$ $[-5, 5]$ by $[-3, 3]$

vertical asymptote is $x = 0$

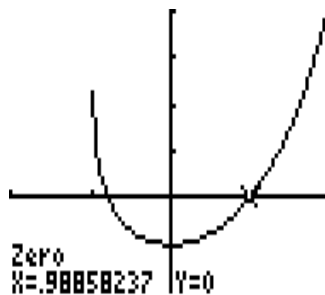
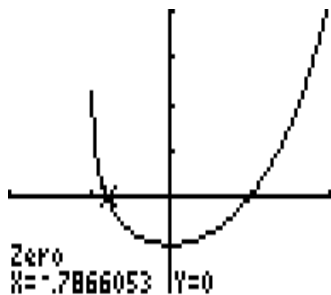


5. (a) $f(x) = g(x)$ for $x = -.79, .99$ (b) $f(x) < g(x)$ for $(-.79, .99)$

$f(x) = y_1 = e^x - 2$ and $g(x) = y_2 = \ln(x + 1)$

x-intercept method Need $y_1 - y_2 = 0$ and $y_1 - y_2 < 0$

$y_1 = e^x - 2 - \ln(x + 1)$ $[-2, 2]$ by $[-2, 4]$

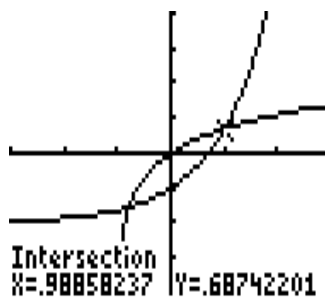
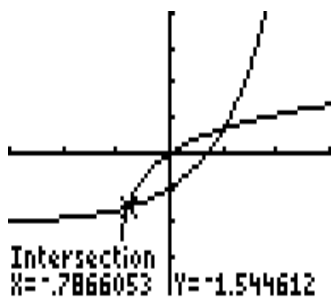


x-intercepts are $x = -.7866053$ and $x = .98858237$

$y_1 - y_2 < 0$ when x is between $-.7866053$ and $.98858237$

points of intersection method Need $y_1 = y_2$ and $y_1 < y_2$

$y_1 = e^x - 2, y_2 = \ln(x + 1)$ $[-3, 3]$ by $[-4, 4]$

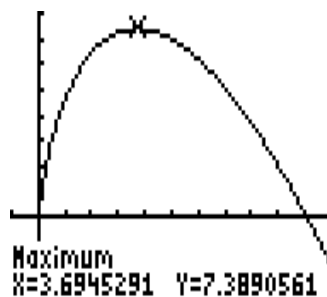


graphs intersect at $x = -.7866053$ and $x = .98858237$

$y_1 < y_2$ when x is between $-.7866053$ and $.98858237$

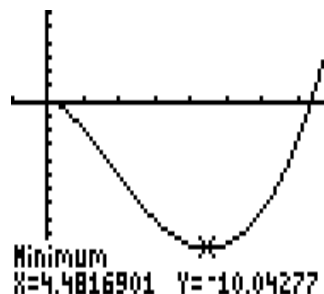
7. (a) $y_1 = 6x - 2x \ln(2x)$
 $[-1, 11]$ by $[-3, 8]$

$y = 7.3890561$, maximum value is 7.39
 $x = 3.6945291$
 increasing $(0, 3.69]$ decreasing $[3.69, \infty)$



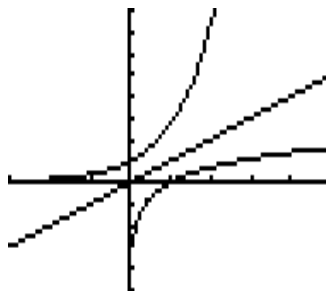
(b) $y_1 = x^2(2 \ln(x) - 2)$
 $[-1, 8]$ by $[-11, 6]$

$y = -10.04277$, minimum value is -10.04
 $x = 4.4816901$,
 increasing $[4.48, \infty)$ decreasing $(0, 4.48]$



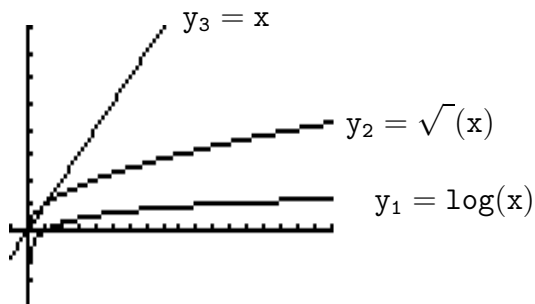
9. $g(x)$ is the reflection of $f(x)$ about the line $y = x$; thus, $f(x)$ and $g(x)$ are inverses of each other.

$y_1 = e^x$, $y_2 = \ln(x)$
 $[-3, 5]$ by $[-5, 8]$



11. $f(x) = y_1$, $g(x) = y_2$, $h(x) = y_3$

$y_1 = \log(x)$, $y_2 = \sqrt{x}$, $y_3 = x$
 $[-1, 18]$ by $[-3, 8]$



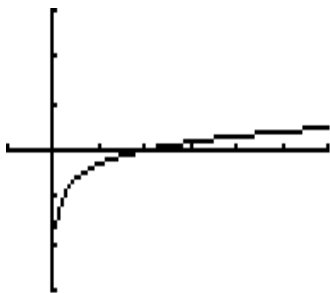
So, $h(x)$, $g(x)$, $f(x)$ in order largest to smallest.

Section 4.3

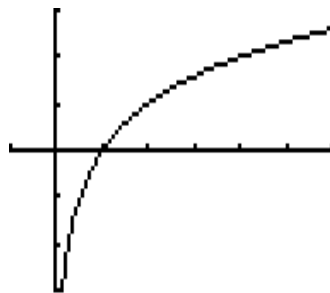
1. $\log\left(\frac{x}{2}\right) = \log(x) - \log(2)$

$[-1, 6]$ by $[-3, 3]$

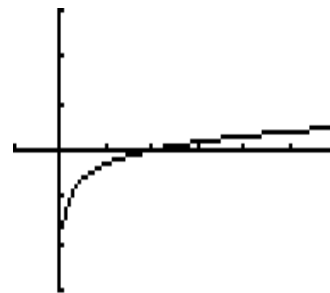
$y_1 = \log(x/2)$



$y_1 = \log(x)/\log(2)$



$y_1 = \log(x) - \log(2)$

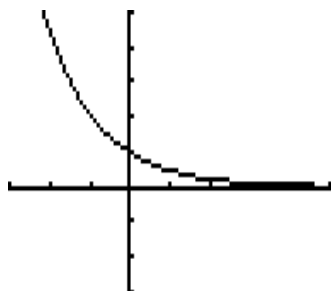


3. They are not the same, so $(\log(3))^x \neq x \log(3)$

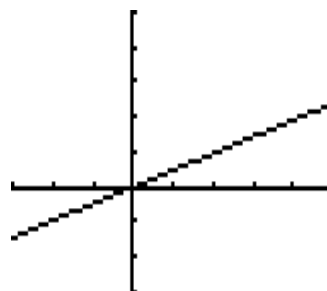
$[-3, 5]$ by $[-3, 5]$

$y_1 = (\log(3)) \wedge x$ is interpreted the same way as $y_1 = \log(3) \wedge x$ on the calculator. The exponent applies to the entire quantity. If we want $\log(3^x)$, we must place the power inside the parentheses like this $y_1 = \log(3 \wedge x)$.

$y_1 = \log(3) \wedge x$



$y_1 = x \log(3)$



The x in $(\log(3))^x$ applies to the whole parentheses - the whole logarithm.

$\log(3)$ is a constant, so $y = x \log(3)$ is x times a constant. Thus, we have a straight line.

Section 4.4

1. solution $(2.6420, \infty)$

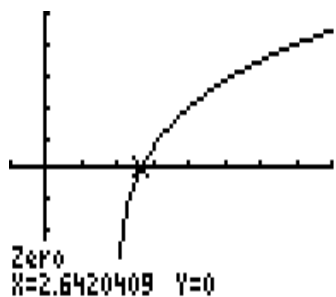
$[-1, 8]$ by $[-4, 5]$

x -intercept method

Need $\ln(x - 2) + \ln(x^2) - 1.5 = 0$

then $\ln(x - 2) + \ln(x^2) - 1.5 > 0$

$y_1 = \ln(x - 2) + \ln(x^2) - 1.5$

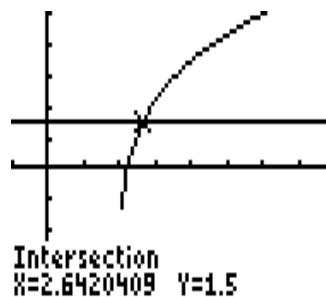


points of intersection method

Need $\ln(x - 2) + \ln(x^2) = 1.5$

then $\ln(x - 2) + \ln(x^2) > 1.5$

$y_1 = \ln(x - 2) + \ln(x^2), y_2 = 1.5$



The x -intercept, or the x -value of the point of intersection, is 2.6420409.

Thus, we want all $x > 2.6420409$.

1. On paper:

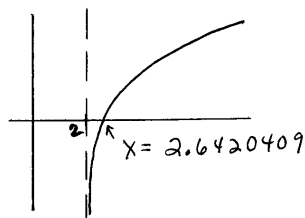
$\ln(x-2) + \ln(x^2) > 1.5$

x -intercept method

$\ln(x-2) + \ln(x^2) - 1.5 > 0$

$y_1 = \ln(x-2) + \ln(x^2) - 1.5$

$[-1, 8]$ by $[-4, 5]$



$y_1 > 0$ for $(2.6420, \infty)$

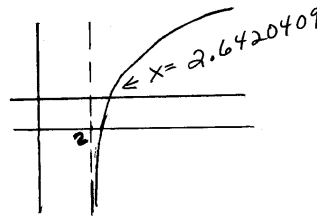
Solution $(2.6420, \infty)$

points of intersection method

$y_1 = \ln(x-2) + \ln(x^2)$

$y_2 = 1.5$

$[-1, 8]$ by $[-4, 5]$



$y_1 > y_2$ for $(2.6420, \infty)$

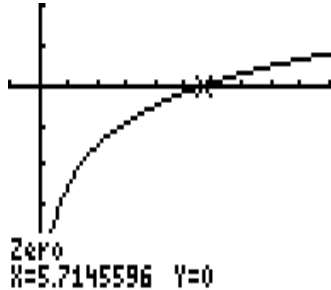
3. solution $x = 5.7146$

x -intercept method

Need $\ln(x) + \log(x) - 2.5 = 0$

$y_1 = \ln(x) + \log(x) - 2.5$

$[-1, 10]$ by $[-5, 2]$

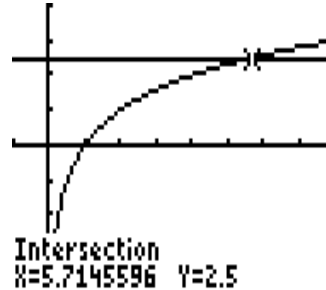


points of intersection method

Need $\ln(x) + \log(x) = 2.5$

$y_1 = \ln(x) + \log(x), y_2 = 2.5$

$[-1, 8]$ by $[-4, 4]$



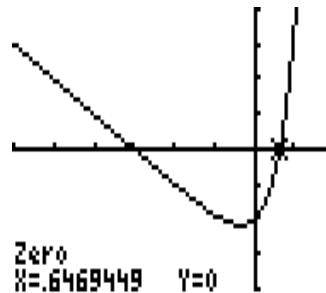
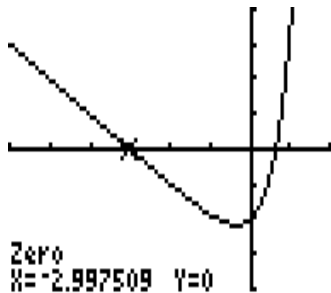
The x -intercept, or the x -value for the point of intersection, is 5.7145596.

5. solutions $x = -2.9975, .6469$

x -intercept method

Need $e^{2x} - x - 3 = 0$

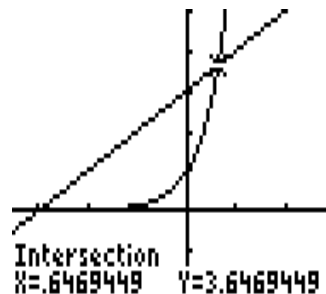
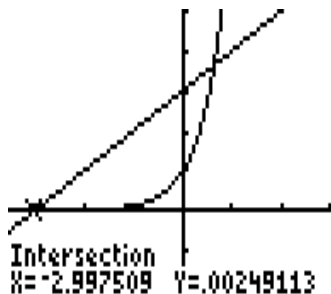
$y_1 = e^{2x} - x - 3$ $[-6, 2]$ by $[-4, 4]$



points of intersection method

Need $e^{2x} = x + 3$

$y_1 = e^{2x}, y_2 = x + 3$ $[-3.5, 3]$ by $[-2, 5]$



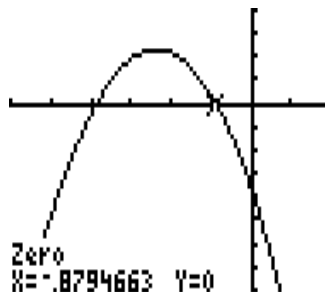
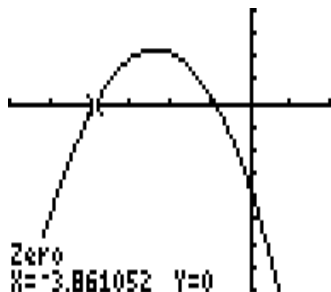
The x -intercepts, or x -values for the points of intersection, are -2.997509 and $.6469449$.

7. solutions $x = -3.8611, -0.8795$

x -intercept method

Need $\ln(1.4 - x) - (x + 2.23)^2 + 1 = 0$

$y_1 = \ln(1.4 - x) - (x + 2.23)^2 + 1$ $[-6, 2]$ by $[-8, 4]$

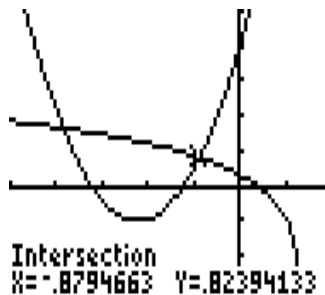
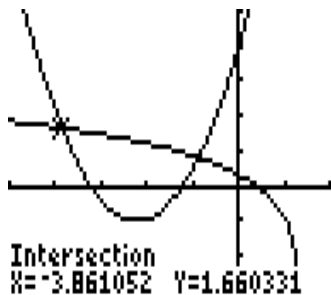


The x -intercepts are -3.861052 and -0.8794663 .

points of intersection method

Need $\ln(1.4 - x) = (x + 2.23)^2 - 1$

$y_1 = \ln(1.4 - x)$, $y_2 = (x + 2.23)^2 - 1$ $[-5, 2]$ by $[-3, 5]$



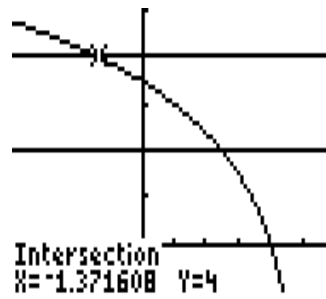
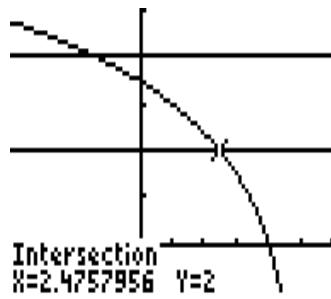
The x -values for the points of intersection are -3.861052 and -0.8794663 .

9. solution $[-1.3716, 2.4758]$

Solve $2 \leq 2.16 \ln(5 - x) \leq 4$

$y_1 = 2$, $y_2 = 2.16 \ln(5 - x)$, $y_3 = 4$ $[-4, 6]$ by $[-1, 5]$

Need all x for which $y_1 \leq y_2 \leq y_3$



The x -values for which the graph of y_2 is between y_1 and y_3 are $[-1.371608, 2.4757956]$,